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And the entire moment when moving with proper angular velocity is

$$\int_{\frac{\pi}{2}}^{z_1} -\frac{8}{9} f v^2 \pi r^3 \tan^3 z_1 \frac{\cos^2 z dz}{\sin^3 z} = \frac{4}{9} \pi f v^2 r^3 \tan^3 z_1 \left( \frac{\cos z_1}{\sin^2 z_1} + \log \tan \frac{1}{2} z_1 \right).$$

Hence,  $\frac{\frac{4}{9} f v^2 r \tan z_1 [(\cos z_1 \div \sin^2 z_1) + \log \tan \frac{1}{2} z_1]}{(\cos z_1 \div \sin^2 z_1) - \log \tan \frac{1}{2} z_1}$  = a maximum for  $z_1$ , (2)

will give the position of wings when the work being done is a maximum.

Therefore, placing the first differential coefficient = 0, and solving for  $\log \tan \frac{1}{2} z_1$ , there results

$$\log \tan \frac{1}{2} z_1 = (\cos z_1 \div \sin^2 z_1) [1 + \cos^2 z_1 \pm \sqrt{(4 + \cos^4 z_1)}].$$

Substituting in (2) this second value, the only one applicable,

$$\frac{\tan z_1 [1 + 1 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{1 - 1 - \cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}} = \frac{\tan z_1 [2 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{\sqrt{(4 + \cos^4 z_1)} - \cos^2 z_1}$$

$$= -\tan z_1 + \frac{1}{2} \tan z_1 [\cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}] = \text{maximum for } z_1.$$

Differentiating a second time and reducing,

$$\cos^6 z_1 - \frac{7}{2} \cos^4 z_1 - 2 \cos^2 z_1 + 2 = 0, \text{ from which}$$

$$\cos^2 z_1 = 0.55154; \therefore z_1 = 42^\circ 2\frac{1}{2}'.$$

### SOLUTION OF A PROBLEM.

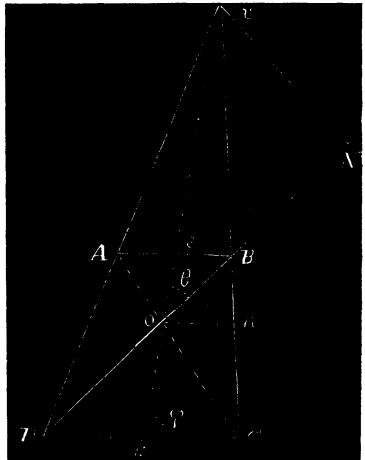
BY PROF. C. M. WOODWARD, WASHINGTON UNIV., ST. LOUIS, MO.

*Problem.*—Given a frustrum of an oblique cone with a circular base; the frustrum is cut in two by a plane perpendicular to the principal plane of the cone, and tangent to the two bases. Find the ratio of the volumes of the two parts of the frustrum.

*Solution.*—Let  $vABCD$  be the section of the cone made by the principal plane, and let  $DBN$  be the trace of the intersecting plane perpendicular to it. The section, projected in  $DB$ , is obviously an ellipse of which  $DB$  is the major axis. Let  $ov = h$ ,  $As = r$ ,  $Dk = R$  and  $on = x$ . Then

$$DB = (R + r) \frac{\sin \varphi}{\sin \theta}.$$

The minor axis is the chord of a circle whose radius is  $\frac{1}{2}(R + r)$ , and whose dist. from the center is  $\frac{1}{2}(R - r)$ ; its length is therefore  $2\sqrt{(Rr)}$ .



The volume of the cone  $v-DB$  is

$$V_0 = \pi \frac{R+r}{2} \frac{\sin \varphi}{\sin \theta} (Rr)^{\frac{1}{2}} \frac{h \sin \theta}{3} = \frac{1}{3} \pi h \sin \varphi \frac{R+r}{2} (Rr)^{\frac{1}{2}},$$

but  $\frac{R+r}{2} = \frac{Rr}{x}$ ; hence  $V_0 = \frac{1}{3x} \pi h \sin \varphi R^{\frac{3}{2}} r^{\frac{3}{2}}$ .

Now  $vs = \frac{hr}{x}$  and  $vk = \frac{hR}{x}$ , hence

cone  $v-AB = V_1 = \frac{1}{3x} \pi h \sin \varphi r^3$ , cone  $v-CD = V_2 = \frac{1}{3x} \pi h \sin \varphi R^3$ .

$$\text{Vol. } ABD = V_0 - V_1 = \frac{1}{3x} \pi h \sin \varphi \left( R^{\frac{3}{2}} - r^{\frac{3}{2}} \right) r^{\frac{3}{2}},$$

$$\text{Vol. } BDC = V_2 - V_0 = \frac{1}{3x} \pi h \sin \varphi \left( R^{\frac{3}{2}} - r^{\frac{3}{2}} \right) R^{\frac{3}{2}},$$

hence  $\frac{\text{Vol. } ABD}{\text{Vol. } BDC} = \frac{\sqrt[3]{r^3}}{\sqrt[3]{R^3}}$ .

From the volumes of the three cones we see that the elliptical cone is a mean proportional between the other two. (This striking analogy between the cones  $v-AB$ ,  $v-BD$ ,  $v-CD$ , and the triangles  $AvB$ ,  $DvB$ ,  $DvC$ , I had never before noticed, though it must have been well known. Neither had I ever observed that the semi-conjugate axis of *such* a conic section is a mean proportional between the radii of the bases of the frustum.)

It is obvious that the plane  $AC$  divides the frustum in the same ratio as the plane  $BD$ .

If the altitude of the frustum is  $p$  we have  $h \sin \varphi \div x = p \div (R-r)$  and

$$V_0 = \frac{1}{3} \pi p [\sqrt[3]{(R^3 r^3) \div (R-r)}],$$

$$V_1 = \frac{1}{3} \pi p [r^3 \div (R-r)], \quad V_2 = \frac{1}{3} \pi p [R^3 \div (R-r)],$$

$$V_0 - V_1 = \frac{1}{3} \pi p \frac{\sqrt[3]{R^3 - \sqrt[3]{r^3}}}{R-r} r^{\frac{3}{2}},$$

$$V_2 - V_0 = \frac{1}{3} \pi p \frac{\sqrt[3]{R^3 - \sqrt[3]{r^3}}}{R-r} R^{\frac{3}{2}}.$$

### SOLUTION OF PROBLEM 255.

BY PROF. W. W. HENDRICKSON, NAVAL ACADEMY, ANNAPOLIS, MD.

Denoting the distance  $AB$  by  $a$ , and taking the axes as represented in the figure, the equations to the lines  $AC$  and  $BC$  are (1.)  $y = x \tan(\varphi + \alpha)$ ,